

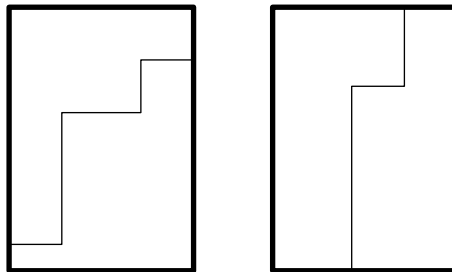
HMMT November 2014
Saturday 15 November 2014
Team Round

Mins and Maxes

1. [3] What is the smallest positive integer n which cannot be written in any of the following forms?
 - $n = 1 + 2 + \cdots + k$ for a positive integer k .
 - $n = p^k$ for a prime number p and integer k .
 - $n = p + 1$ for a prime number p .
2. [5] Let $f(x) = x^2 + 6x + 7$. Determine the smallest possible value of $f(f(f(f(x))))$ over all real numbers x .
3. [5] The side lengths of a triangle are distinct positive integers. One of the side lengths is a multiple of 42, and another is a multiple of 72. What is the minimum possible length of the third side?

Enumeration

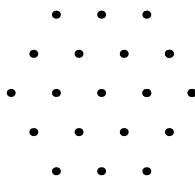
4. [3] How many ways are there to color the vertices of a triangle red, green, blue, or yellow such that no two vertices have the same color? Rotations and reflections are considered distinct.
5. [5] Let A, B, C, D, E be five points on a circle; some segments are drawn between the points so that each of the $\binom{5}{2} = 10$ pairs of points is connected by either zero or one segments. Determine the number of sets of segments that can be drawn such that:
 - It is possible to travel from any of the five points to any other of the five points along drawn segments.
 - It is possible to divide the five points into two nonempty sets S and T such that each segment has one endpoint in S and the other endpoint in T .
6. [6] Find the number of strictly increasing sequences of nonnegative integers with the following properties:
 - The first term is 0 and the last term is 12. In particular, the sequence has at least two terms.
 - Among any two consecutive terms, exactly one of them is even.
7. [7] Sammy has a wooden board, shaped as a rectangle with length 2^{2014} and height 3^{2014} . The board is divided into a grid of unit squares. A termite starts at either the left or bottom edge of the rectangle, and walks along the gridlines by moving either to the right or upwards, until it reaches an edge opposite the one from which the termite started. Depicted below are two possible paths of the termite.



The termite's path dissects the board into two parts. Sammy is surprised to find that he can still arrange the pieces to form a new rectangle not congruent to the original rectangle. This rectangle has perimeter P . How many possible values of P are there?

Hexagons

8. [3] Let \mathcal{H} be a regular hexagon with side length one. Peter picks a point P uniformly and at random within \mathcal{H} , then draws the largest circle with center P that is contained in \mathcal{H} . What is this probability that the radius of this circle is less than $\frac{1}{2}$?
9. [5] How many lines pass through exactly two points in the following hexagonal grid?



10. [8] Let $ABCDEF$ be a convex hexagon with the following properties.
- (a) \overline{AC} and \overline{AE} trisect $\angle BAF$.
 - (b) $\overline{BE} \parallel \overline{CD}$ and $\overline{CF} \parallel \overline{DE}$.
 - (c) $AB = 2AC = 4AE = 8AF$.

Suppose that quadrilaterals $ACDE$ and $ADEF$ have area 2014 and 1400, respectively. Find the area of quadrilateral $ABCD$.